



# High School Math Materials

School Board Work Study Session

June 10, 2009

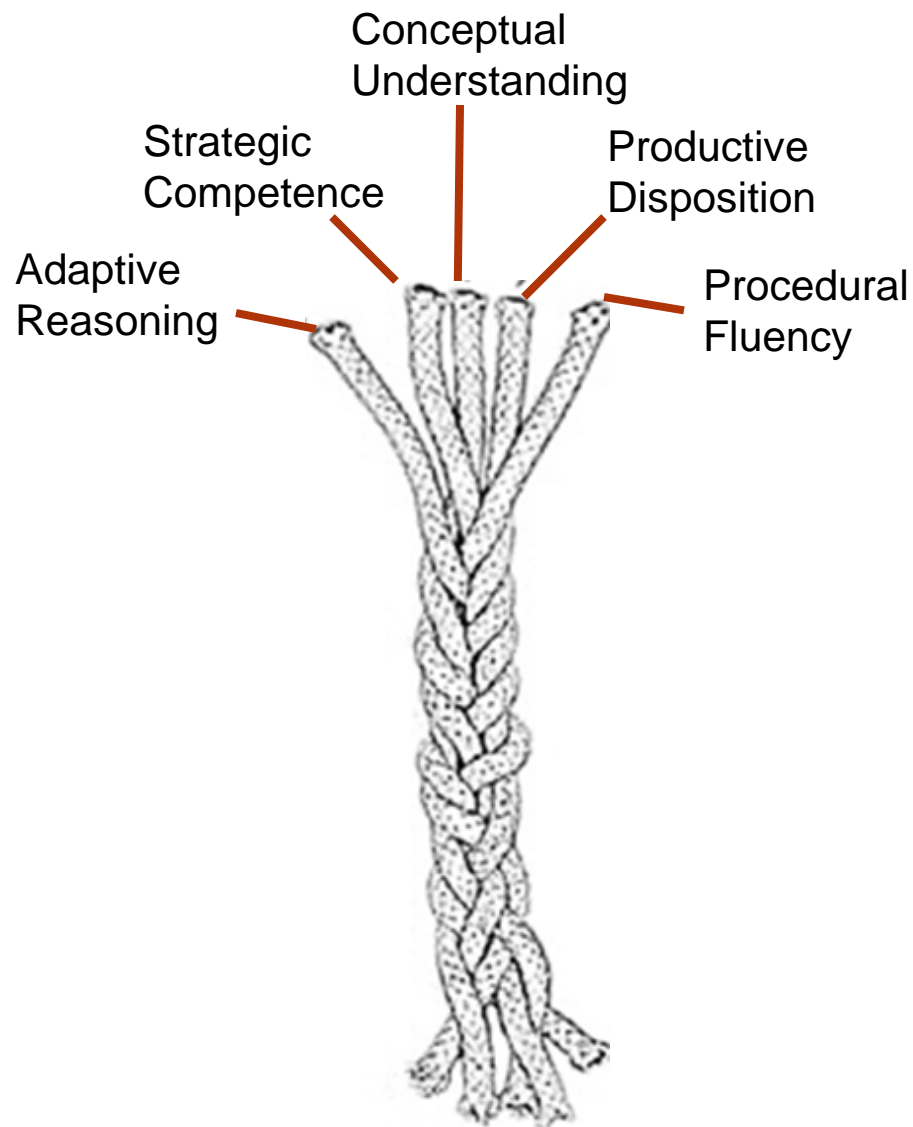


# Agenda:

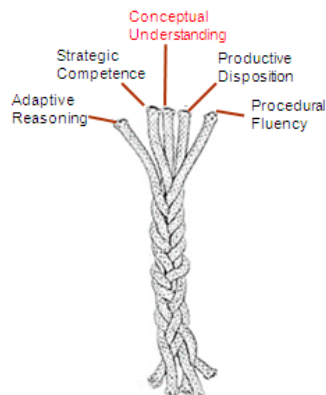
- Process
- Materials
- Dialog with our teachers

# HS Math Materials - Vision

Our goal is for all students to be mathematically prepared and confident to pursue higher education or specialized career training, and to be equipped to apply mathematics as a life tool. To accomplish this goal, every student will become mathematically proficient, as defined by the National Research Council, to include the following five interwoven strands:



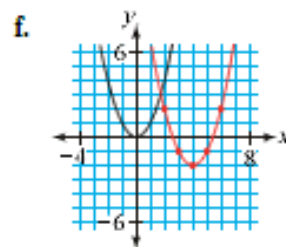
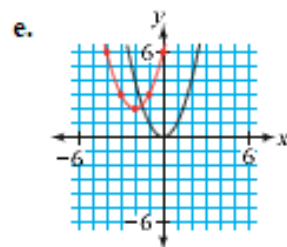
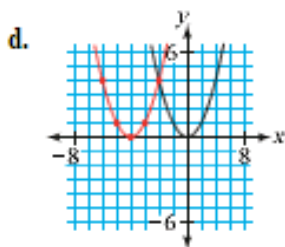
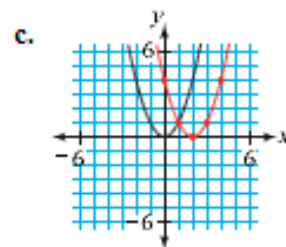
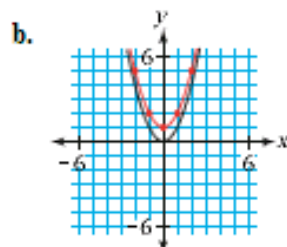
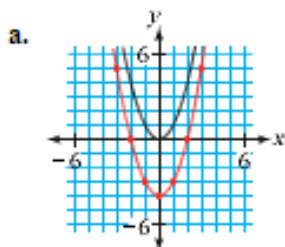
Intertwined Strands of Proficiency



# Conceptual Understanding

Step 1

Each graph below shows the graph of the parent function  $y = x^2$  in black. Find a quadratic equation that produces the congruent, red parabola. Apply what you learned about translations of the graphs of linear equations in Lesson 4.3.



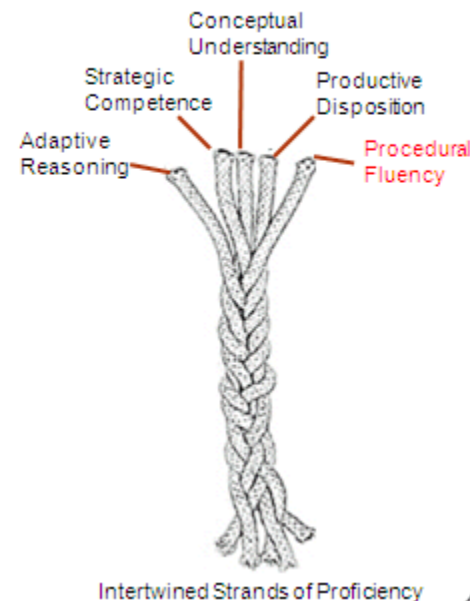
Step 2

Write a few sentences describing any connections you discovered between the graphs of the translated parabolas, the equation for the translated parabola, and the equation of the parent function  $y = x^2$ .

Step 3

In general, what is the equation of the parabola formed when the graph of  $y = x^2$  is translated horizontally  $h$  units and vertically  $k$  units?

# Procedural Fluency



## Lesson 9.7 • Graphs of Rational Functions

Name \_\_\_\_\_ Period \_\_\_\_\_ Date \_\_\_\_\_

1. Rewrite each rational expression in factored form.

a.  $\frac{x^2 - 5x - 6}{x^2 - 25}$

b.  $\frac{2x^2 + 3x - 2}{3x^2 - 5x - 2}$

c.  $\frac{x^2 - 16}{6x^2 - 7x - 3}$

d.  $\frac{4x^3 - 25x}{x^2 - 14x + 48}$

e.  $\frac{x^3 + 5x^2 - 24x}{x^2 + 6x + 9}$

f.  $\frac{9x^2 - 1}{2x^3 - x^2 - 3x}$

2. Rewrite each expression in rational form (as the quotient of two polynomials).

a.  $\frac{2}{x} + 3$

b.  $1 + \frac{1}{x - 2}$

c.  $4 + \frac{2x - 7}{x + 5}$

d.  $\frac{3x + 4}{2x - 3} - 1$

e.  $\frac{5x - 7}{x + 3} - 4$

f.  $-6 + \frac{10x - 3}{3x - 5}$

3. Find all vertical and horizontal asymptotes of the graph of each rational function.

a.  $f(x) = \frac{x}{x - 1}$

b.  $f(x) = \frac{2x + 5}{x + 3}$

c.  $f(x) = -\frac{1}{x^2}$

d.  $f(x) = \frac{3}{(x - 2)^2}$

e.  $f(x) = \frac{x^2 + x + 1}{x^2 - 4}$

f.  $f(x) = \frac{x - 3}{x^2 + 6x + 8}$

4. Find all vertical and slant asymptotes of the graph of each rational function.

a.  $f(x) = \frac{x^2}{x - 1}$

b.  $f(x) = \frac{x^2 + 1}{x}$

c.  $f(x) = \frac{x^2 + x - 1}{x - 1}$

d.  $f(x) = \frac{2x^2 - 5}{x + 3}$

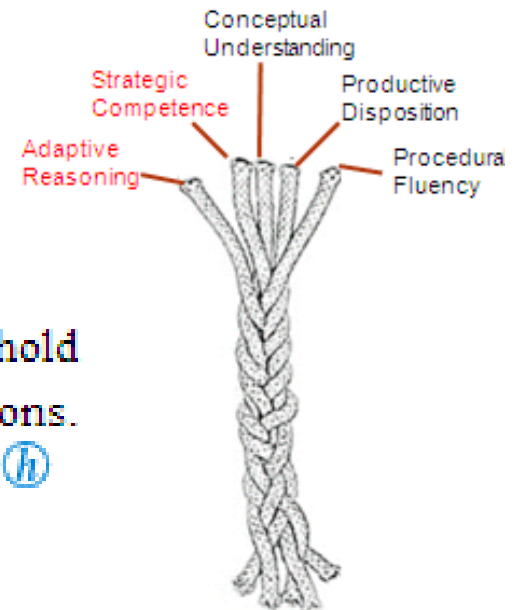
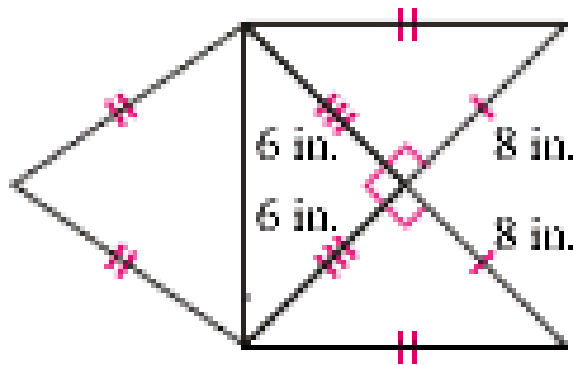
e.  $f(x) = \frac{x^3}{x^2 - 4}$

f.  $f(x) = \frac{9 - x^2}{2 + x}$

5. Give the coordinates of all holes in the graph of each rational

# Problem Solving

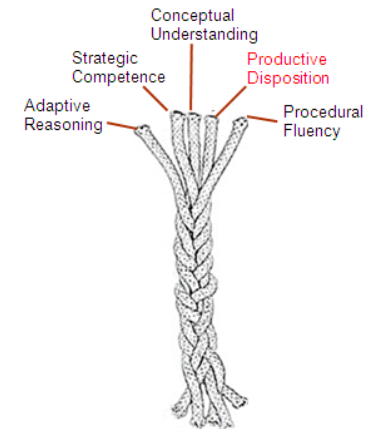
15. Jamala has designed a container that she claims will hold  $50 \text{ in.}^3$ . The net is shown at right. Check her calculations. What is the volume of the solid formed by this net?  $\textcircled{h}$



Intertwined Strands of Proficiency

# Productive Disposition

CHAPTER 6 REVIEW • CHAPTER 6 REVIEW • CHAPTER 6 REVIEW • CHAPTER 6 REVIEW

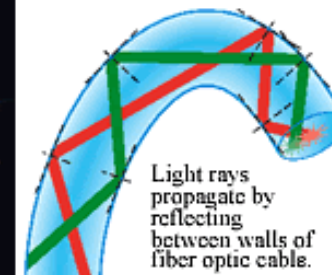


Intertwined Strands of Proficiency

16. **APPLICATION** Suppose the signal strength in a fiber-optic cable diminishes by 15% every 10 mi.
- What percentage of the original signal strength is left after a stretch of 10 mi?
  - Create a table of the percentage of signal strength remaining in 10 mi intervals, and make a graph of the sequence.
  - If a phone company plans to boost the signal just before it falls to 1%, how far apart should the company place its booster stations?

## Technology CONNECTION

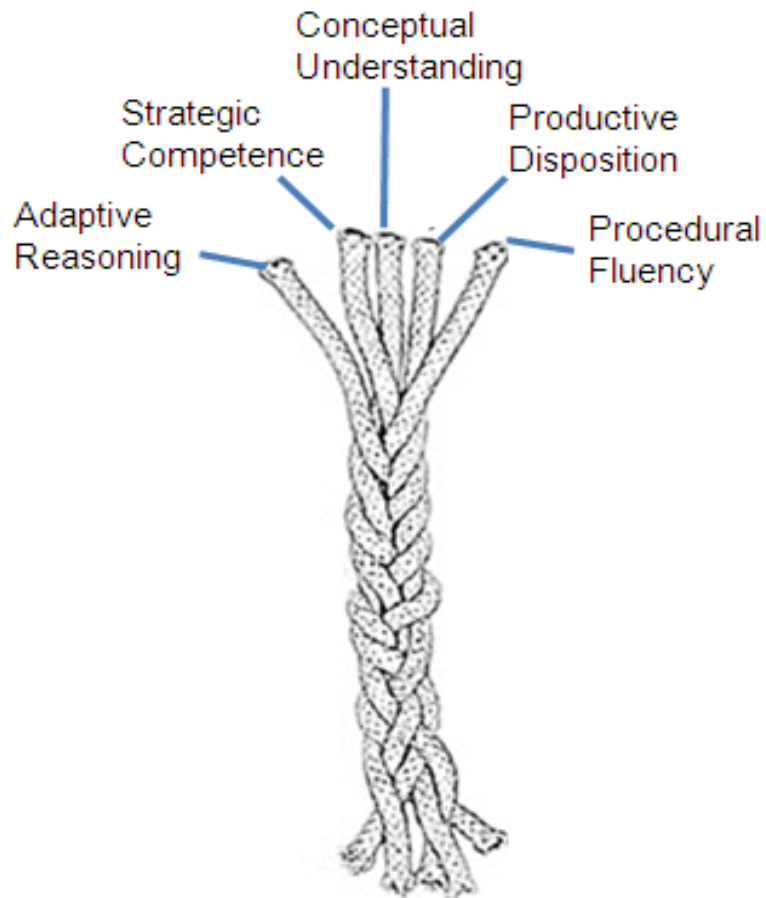
Fiber-optic technology uses light pulses to transmit information from one transmitter to another down fiber lines made of silica (glass). Fiber-optic strands are used in telephone wires, cable television lines, power systems, and other communications. These strands operate on the principle of total internal reflection, which means that the light pulses cannot escape out of the glass tube and instead bounce information from transmitter to transmitter.



Light rays propagate by reflecting between walls of fiber optic cable.

The photo on the left shows strands of fiber-optic cable. The illustration on the right shows how light is reflected along a strand of fiber-optic cable, creating total internal reflection.

# HS Math Materials - Vision



Intertwined Strands of Proficiency

- *Conceptual understanding*
- *Procedural fluency*
- *Strategic competence*
- *Adaptive reasoning*
- *Productive disposition*

# Process:

## High School Math Materials Workgroup

- IHS

- Rochelle Eixenberger
- Jon Ko
- Ben Reed

- LHS

- Randall Imes
- Angie Kruzich
- Michael Snow

TMCHS

Hilary Oliver

- SHS

- Kerry Dunbar
- Ruth Edwards
- Amie Karkainen

- PCFC

- Jodi Carter
- Jeff Glover
- Christina Morris

- Principal Representation

- Math Specialist



# Charter

Focus on student achievement as the paramount priority.



# Charter Focus:



- **Board's Mission**
- **All Students**
- **Cultural Competence**
- **Curriculum Coherence**
- **Articulated Continuity**
- **On-going Assessment Focus**
- **High Leverage Practices**
- **Standards Based**

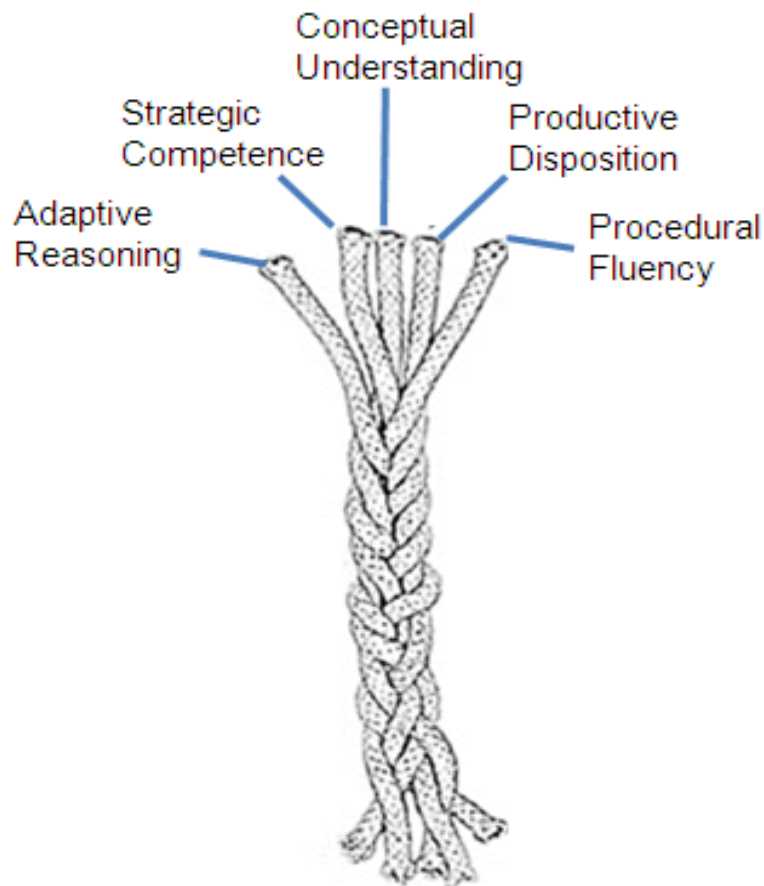
# WA State Standards:

An effective mathematics program balances three important components of mathematics:

- Conceptual understanding (making sense of mathematics)
- Procedural proficiency (skills, facts and procedures)
- Problem solving and mathematical processes (using mathematics to reason, think and apply mathematical knowledge)

“These standards make clear the importance of all three of these components, purposefully interwoven to support students’ development as increasingly sophisticated mathematical thinkers. The standards are written to support the development of students so that they know and understand mathematics.” (page v.)

# HS Math Materials - Vision



Intertwined Strands of Proficiency

- *Conceptual understanding*
- *Procedural fluency*
- *Strategic competence*
- *Adaptive reasoning*
- *Productive disposition*

# The Process

- Developed rubrics, based on Vision, for
  - Initial Screening
  - In-Depth Screening
- Screened eight sets of materials with initial screening tool:
  - Top materials from the State Core Materials Review
  - Materials that followed our Middle School and Elementary materials
- Choose three sets of materials for in-depth screening:
  - Discovering Mathematics
  - Holt Mathematics
  - Prentice Hall Mathematics

# In-depth Screening

- After screening with the In-depth screening rubric, the group decided to continue to examine two sets of materials:
  - Discovering Mathematics
  - Holt Mathematics
- Started Intensive Review

# Intensive Review:

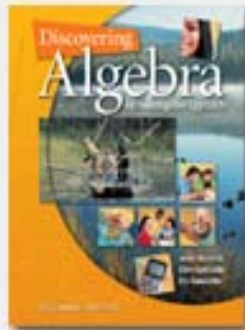
- Lesson planning and working through the problems in parallel lessons from both programs
- Teaching lessons in classrooms and collecting student feedback
- Analyzing how the standards are taught in both programs
- Careful examination of trigonometry
- Expert Panels with set questions
- Using the goals of the charter to examine the programs
- Answering the question for each program: What would it take for this program to meet the goals given us by the Charter?

# Unanimous Recommendation: Key Curriculum Press

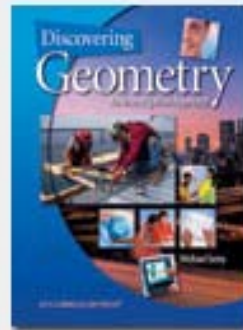
Discovering Algebra

Discovering Geometry

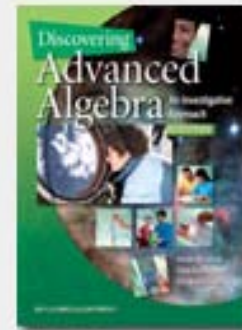
Discovering Advanced Algebra



Discovering Algebra



Discovering Geometry



Discovering Advanced  
Algebra

# Why *Discovering Mathematics*?

Because it has a balance of:

- Lots of practice problems and the ability to generate more practice using resources within the program
- Investigations so students develop conceptual understanding
- Formalization of concepts using the words, numbers and symbols of mathematics

# Text books – The Distributive Property

- Links what students have learned before to the topic they are about to discuss



## Equivalent Algebraic Equations

In Lesson 4.3, you learned how to find an equation of a line through a given point. But a line goes through many points, so if you choose a different point, you'll get a different equation! In this lesson, you'll learn how to identify different equations that describe the same line.



# Text books – The Distributive Property

- Defines and describes explicitly what they will be learning:

There are many different **equivalent equations** that can be used to describe any given line. In fact, both of the equations above can also be described in intercept form,  $y = a + bx$ . In this lesson you'll learn how to change equations to equivalent equations in intercept form by using mathematical properties and the rules for order of operations.

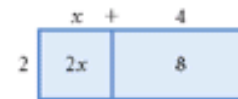
# Text books – The Distributive Property

- Provides an example showing students what to do and why

**EXAMPLE A** | Use the distributive property to write  $y = 3 + 2(x + 4)$  without parentheses.

► **Solution**

Before adding 3, distribute the 2 through the sum of  $x$  and 4.



$$y = 3 + 2(x + 4)$$

Point-slope equation.

$$y = 3 + 2 \cdot x + 2 \cdot 4$$

Use the distributive property: Distribute 2 through  $x + 4$ .

$$y = 3 + 2x + 8$$

Multiply  $2 \cdot 4$ .

$$y = 11 + 2x$$

Combine like terms (add  $3 + 8$ ).

So,  $y = 3 + 2(x + 4)$  is equivalent to  $y = 11 + 2x$ . These are a point-slope equation and an intercept equation for the same line. What does each of the forms tell you about the line it describes?

# Text books – The Distributive Property

- Generalizes

The distributive property can be generalized like this:

## **Distributive Property**

For any values of  $a$ ,  $b$ , and  $c$ , this equation is true:

$$a(b + c) = a \cdot b + a \cdot c$$

# Text books – The Distributive Property

- Provides an investigation to solidify student understanding

In the investigation you'll further explore how to identify equivalent equations.



## Investigation Equivalent Equations

Here are six different-looking equations in point-slope form.

a.  $y = 3 - 2(x - 1)$

b.  $y = -5 - 2(x - 5)$

c.  $y = 9 - 2(x + 2)$

d.  $y = 0 - 2(x - 2.5)$

e.  $y = 7 - 2(x + 1)$

f.  $y = -9 - 2(x - 7)$

- |        |   |
|--------|---|
| Step 1 | Do the six equations represent the same line or different lines? Explain.   |
| Step 2 | Divide these equations among the members of your group. Use the distributive property to rewrite the right side of each equation. When you combine like terms, you should get an equation in intercept form.    |
| Step 3 | Enter your point-slope equation into Y1, and enter your intercept equation into Y2. Check that the two equations have the same calculator graph or table. How does this show that the equations are equivalent? |

# Text books – The Distributive Property

- Summarizes
- 



You have learned how to write linear equations in two different forms:

Intercept form  $y = a + bx$

Point-slope form  $y = y_1 + b(x - x_1)$

In the second part of the investigation, some of the equations had  $x$  and  $y$  on the same side, as in  $12x + 2y = -6$ . Equations in the form  $ax + by = c$  are in **standard form**. What other equation in the investigation is in standard form?

No matter what form you start with, you can always rewrite any linear equation in intercept form. Then it's easy to recognize equivalent equations. Let's review properties that help you change the form of an equation.

# Text books – The Distributive Property

- Provide practice for procedural fluency:

## EXERCISES

Use your graphing calculator for Exercises 1, 2, and 10.



### ▶ Practice Your Skills

1. Is each pair of expressions equivalent? If they are not, change the second expression so that they are equivalent. Check your work on your calculator by comparing table values when you enter the equivalent expressions into Y1 and Y2.
  - a.  $3 - 3(x + 4)$        $3x - 9$  @
  - b.  $5 + 2(x - 2)$        $2x + 1$
  - c.  $5x - 3$        $2 + 5(x - 1)$
  - d.  $-2x - 8$        $-2(x - 4)$

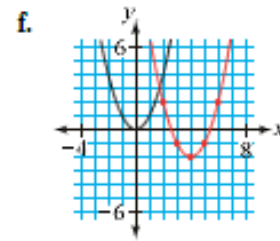
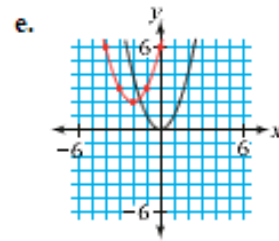
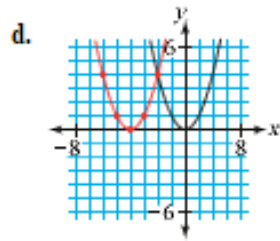
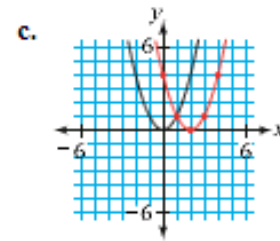
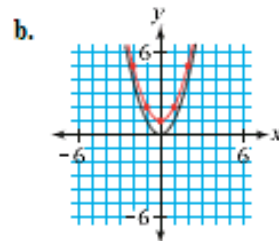
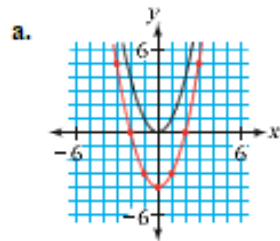
# Text books – The Distributive Property

## ► Reason and Apply

6. Solve each equation for the indicated variable.
- a.  $y = 3(x + 8)$  solve for  $x$
  - b.  $\frac{y - 3}{x - 4} = 10$  solve for  $y$
  - c.  $4(2y - 5) - 12 = x$  solve for  $y$
7. In the expression  $3x + 15$ , the greatest common factor (GCF) of both  $3x$  and  $15$  is  $3$ . You can write the expression  $3x + 15$  as  $3(x + 5)$ . This process, called **factoring**, is the reverse of distributing. Rewrite each expression by factoring out the GCF that will leave  $1$  as the coefficient of  $x$ . Use the distributive property to check your work.
- a.  $3x - 12$  @
  - b.  $-5x + 20$  @
  - c.  $32 + 4x$
  - d.  $-7x - 28$
8. **Mini-Investigation** Consider the equation  $y = 10 + 5x$  in intercept form.
- a. Factor the right side of the equation.
  - b. Use the commutative property of addition to swap the terms inside the parentheses.
  - c. Your result should look similar to the point-slope form of the equation. What's missing? What is the value of this missing piece? @
  - d. What point could you use to write the point-slope equation in 8c? What is special about this point? @
9. In each set of three equations, two equations are equivalent. Find them and explain how you know they are equivalent.
- a. i.  $y = 14 - 2(x - 5)$
  - b. i.  $y = -13 + 4(x + 2)$

# Algebra 2 - Translations and the Quadratic Family

Step 1 Each graph below shows the graph of the parent function  $y = x^2$  in black. Find a quadratic equation that produces the congruent, red parabola. Apply what you learned about translations of the graphs of linear equations in Lesson 4.3.

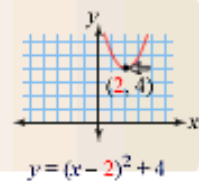
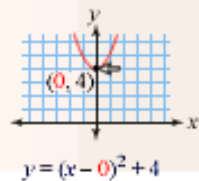
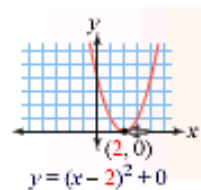
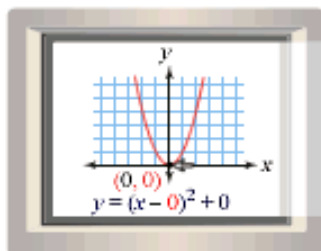
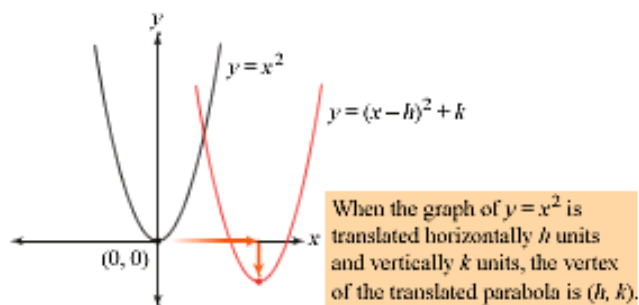


Step 2 Write a few sentences describing any connections you discovered between the graphs of the translated parabolas, the equation for the translated parabola, and the equation of the parent function  $y = x^2$ .

Step 3 In general, what is the equation of the parabola formed when the graph of  $y = x^2$  is translated horizontally  $h$  units and vertically  $k$  units?

# Algebra 2: Formalization

It is important to notice that the vertex of the translated parabola is  $(h, k)$ . That's why finding the vertex is fundamental to determining translations of parabolas. In every function you learn, there will be key points to locate. Finding the relationships between these points and the corresponding points in the parent function enables you to write equations more easily.

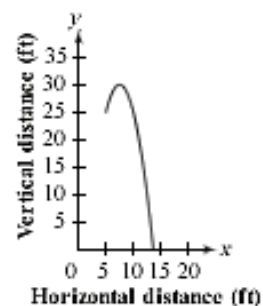


# Algebra 2: Example Problem

## **EXAMPLE**

This graph shows a portion of a parabola. It represents a diver's position (horizontal and vertical distance) from the edge of a pool as he dives from a 5 ft long board 25 ft above the water.

- Sketch a graph of the diver's position if he dives from a 10 ft long board 10 ft above the water. (Assume that he leaves the board at the same angle and with the same force.)
- In the scenario described in part a, what is the diver's position when he reaches his maximum height?



# Algebra 2: Example Solution

## ► **Solution**

First, make sure that you can interpret the graph. The point  $(5, 25)$  represents the moment when the diver leaves the board, which is 5 ft long and 25 ft high. The vertex,  $(7.5, 30)$ , represents the position where the diver's height is at a maximum, or 30 ft; it is also the point where the diver's motion changes from upward to downward. The  $x$ -intercept, approximately  $(13.6, 0)$ , indicates that the diver hits the water at approximately 13.6 ft from the edge of the pool.

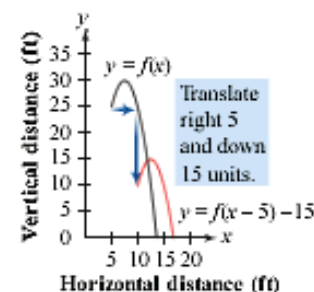
a. If the length of the board increases from 5 ft to 10 ft, then the parabola translates right 5 units. If the height of the board decreases from 25 ft to 10 ft, then the parabola translates down 15 units. If you define the original parabola as the graph of  $y = f(x)$ , then the function for the new graph is  $y = f(x - 5) - 15$ .

b. As with every point on the graph, the vertex translates right 5 units and down 15 units. The new vertex is  $(7.5 + 5, 30 - 15)$ , or  $(12.5, 15)$ . This means that when the diver's horizontal distance

from the edge of the pool is 12.5 ft, he reaches his maximum height of 15 ft.



Mark Ruiz placed first in the 2000 U.S. Olympic Diving Team trials with this dive.

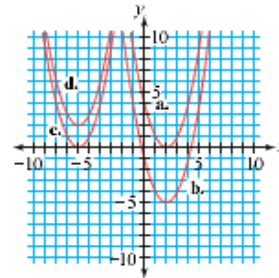
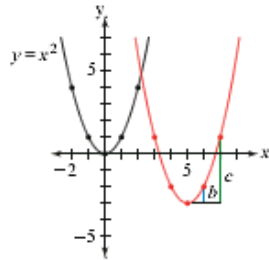


The translations you investigated with linear functions and functions in general work the same way with quadratic functions. If you translate the graph of  $y = x^2$  horizontally  $h$  units and vertically  $k$  units, then the equation of the translated graph is  $y = (x - h)^2 + k$ . You can check

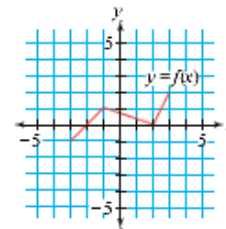
# Practice Problems

## Reason and Apply

6. Write an equation for each parabola at right.
7. The red parabola below is the image of the graph of  $y = x^2$  after a translation right 5 units and down 3 units.



- a. Write an equation for the red parabola.
- b. Where is the vertex of the red parabola?
- c. What are the coordinates of the other four points if they are 1 or 2 horizontal units from the vertex? How are the coordinates of each point on the black parabola related to the coordinates of the corresponding point on the red parabola?
- d. What is the length of blue segment  $b$ ? Of green segment  $c$ ?
8. Given the graph of  $y = f(x)$  at right, draw a graph of each of these related functions.
- a.  $y = f(x + 2)$
- b.  $y = f(x - 1) - 3$



# Additional Resources:

- Complete worked out solutions manual
- Assessment handbook and CD, worksheet builder technology and Standardized Test Preparation Handbook
- Teaching and Worksheet masters

## **Available online or in paper version:**

- Practice Your Skills worksheets
- Condensed Lessons in English and Spanish – additional explanation of each lesson to support students who are absent or need more help
- Parent Guide – gives overview of each chapter

# Key Curriculum Press Online book:

data set is **bimodal**.

3. One hundred  
like the one at  
the circle.

**R**  
radical expressions, 535  
discriminants, 568



- Interactive Glossary Terms
- Selected Answers Links
- Interactive Index Terms
- More Practice your Skills Links

# Key Curriculum Press Online Text book:

[\[> See Calculator Note 1G <\]](#)



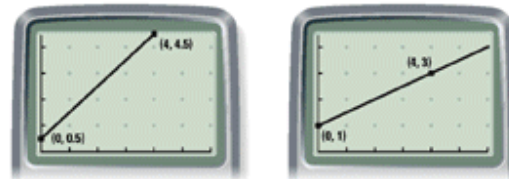
[www.keymath.com/DA](http://www.keymath.com/DA)



- Calculator Note Links
- Chapter-specific Web resources
- Home Page Link
- Fathom Links
- Sketch pad links

# Technology: Graphing Calculators and CBRs

In this investigation you'll analyze time-distance graphs, and you'll use a motion sensor to create your own graphs.

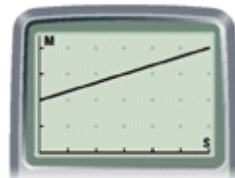


## Investigation Walk the Line

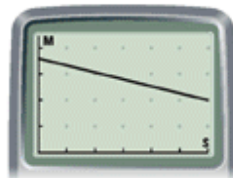
### You will need

- a 4-meter measuring tape or four metersticks per group
- a motion sensor
- a stopwatch or watch that shows seconds

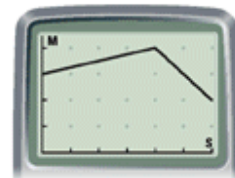
Imagine that you have a 4-meter measuring tape positioned on the floor. A motion sensor measures your distance from the tape's 0-mark as you walk, and it graphs the information. On the calculator graphs shown here, the horizontal axis shows time from 0 to 6 seconds and the vertical axis shows distance from 0 to 4 meters.



a.



b.



c.

# Technology: Geometer's Sketchpad

The Geometer's Sketchpad - [Polygon\_Exterior\_Angles[1] - Exterior Angle Sum Conjecture]

File Edit Display Construct Transform Measure Graph Window Help

Lesson 5.2 Exterior Angle Sum Conjecture

$m\angle JAB = 80^\circ$        $m\angle FBC = 84^\circ$

$m\angle GCD = 41^\circ$

$m\angle HDE = 110^\circ$

$m\angle IEA = 45^\circ$

$m\angle JAB + m\angle FBC + m\angle GCD + m\angle HDE + m\angle IEA = 360.00^\circ$

For any polygon, the sum of the measures of a set of exterior angles is  $360^\circ$ .

Exterior Angle Sum Conjecture | Equiangular Polygon Conjecture

# Technology: Fathom

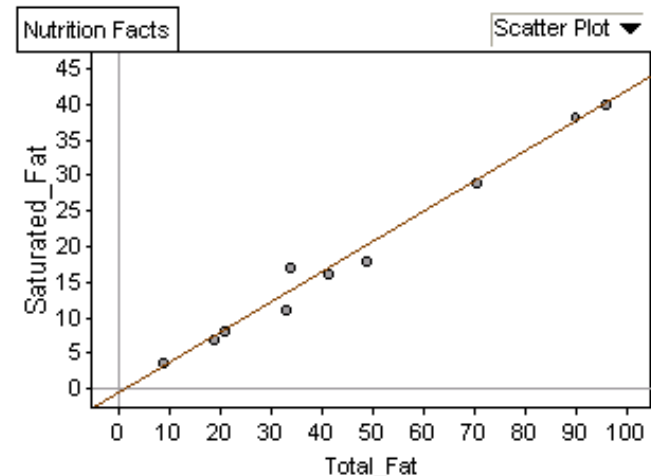
Fathom - [FastFood[1]]

File Edit Display Insert Data Analyze Window Help



Nutrition Facts

	Burger	Saturated_Fat	Total_Fat
1	Burger King Bacon Double Cheesburger	17.0	34.0
2	Burker King Original WHOPPER Sandwich with Cheese	18.0	49.0
3	Hardee's 2/3 lb Double Thickburger	38.0	90.0
4	Hardee's 2/3 lb Bacon Cheese Thickburger	40.0	96.0
5	Jack in the Box Bacon Ultimate Cheeseburger	29.0	70.5
6	Jack in the Box Jumbo Jack w/ Cheese	16.0	41.5
7	McDonald's Big Mac	11.0	33.0
8	McDonald's Quarter Pounder	8.0	21.0
9	Wendy's Jr. Hamburger	3.5	9.0
10	Wendy's Classic Single w/ Everything	7.0	19.0



Questions?

